Mathematics II

029

14 Nov. 2012 **8.30-11.30 am**

REPUBLIC OF RWANDA

RWANDA EDUCATION BOARD (REB)

ADVANCED LEVEL NATIONAL EXAMINATIONS 2012

SUBJECT: MATHEMATICS II

COMBINATIONS:

- **MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)**
- **MATHS-COMPUTER SCIENCE-ECONOMICS (MCE)**
- **MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)**
- **MATHS-PHYSICS-COMPUTER SCIENCE (MPC)**
- **MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)**
- **PHYSICS-CHEMISTRY-MATHEMATICS (PCM)**
- **PHYSICS-ECONOMICS-MATHEMATICS (PEM)**

DURATION: 3 HOURS

INSTRUCTIONS:

This paper consists of **two** sections: **A** and **B**.

Section A: Attempt **all** questions. **(55 marks)**

Section B: Attempt any **three** questions. **(45 marks)**

Geometrical instruments and silent non-programmable calculators may be used.

SECTION A : Attempt all questions. **(55 marks)**

01. Show that C(n-1, p-1)+C(n-1, p) = C(n, p). ***(4 marks)***

02. Find the total number of diagonals that can be drawn in a decagon. ***(3 marks)***

03. Determine the continuity of $f(x)=\frac{\ln x+\tan^{-1}x}{(x-1)(x+1)}$. ***(3 marks)***

04. Find the value of x if $\sqrt{3}\tan x = 2\sin x$ ***(3 marks)***

05. The matrix $M(\alpha)$ is define by

$$M(\alpha)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}.$$

Verify that $M(\alpha)M(\beta)=M(\alpha+\beta)$. ***(2 marks)***

06. A person, standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60^0; when he retreats 40 *meters* from the bank, he finds the angle to be 30^0. Find the breadth of the river and the height of the tree. ***(5 marks)***

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07. If T_p, T_q and T_r are the p^{th}, q^{th} and r^{th} terms of an arithmetic progression , then find the value of $\begin{vmatrix}T_p & T_q & T_r\\ p & q & r\\ 1 & 1 & 1\end{vmatrix}$ ***(3 marks)***

08. For what value of k, the points $(1,5)$, $(k,1)$ and $(11,7)$ are collinear? ***(3 marks)***

? 09. Evaluate $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3}$ ***(3 marks)***

10. From the following data of marks in Mathematics and Physics obtained by four students out of thirty.
Calculate the correlation coefficient : ***(5 marks)***

Mathematics:	14	45	27	38
Physics :	35	40	20	21

11. In Euclidian space IR^2, the sphere with $M(2,-1,3)$ as center passes through the point $T(1,2,-3)$. Write the equation of the sphere and parametric equations of a line which is tangent through T. ***(4 marks)***

12. A tank is the form of an inverted cone having height *8* meters and radius *2* meters. Water is flowing into the tank at the rate of $\frac{1}{8}m^3$/minute. How fast is the water level rising when the water is 2.5 meters deep? ***(4 marks)***

13. Calculate :

a) $\int \frac{\sin x}{1+\sin x}dx$ ***(3 marks)***

b) $\int_0^2 \frac{5x+1}{x^2+4}dx$ ***(3 marks)***

14. a) In a single throw of two dice, determine the probability of getting a total of 2 or 4. ***(2 marks)***

b) The letters of the word "**DIVORCE**" are arranged at random. Find the probability that the vowels may occupy the even places. ***(2 marks)***

15. Find the sum of $1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\cdots$ ***(3 marks)***

SECTION B: Attempt ONLY THREE questions *(45 marks)*

16. Consider a real valued numerical function defined as
$f: IR \to IF$

$x \to \frac{1}{2}x^2e^{x+1}$.

a) Find the domain of function $f(x)$ ***(1 mark)***

b) Find the intersection with axis of coordinates. ***(2 marks)***

c) Find the asymptotes ***(5 marks)***

d) Discuss the first and second derivative of $f(x)$ ***(3 marks)***

e) Sketch the graph of $f(x)$ ***(2 marks)***

17. The sides of perfect die are colored as follows: three sides are orange, two sides are green and one side is red. A player bets 200 RWF is refunded for each throw. When red face of the die is up, a player is refunded 10 % of 200 RWF, when orange face is up, a player is refunded 30 % of 200 RWF and when green face is up, a player is given 500 RWF. If X is the difference between the refunded money and the betted money,

a) determine the sets of values of X and the distribution probability of X. ***(5.5 marks)***

b) calculate the mathematical expectation E(X) of X and interpret the obtained values. **(4 marks)**

c) calculate the variance and the standard deviation of X. **(5.5 marks)**

18. A straight line passes through points $A(-1,-5)$, $B(0,-8)$ and $2y+16=4x^2$ is the equation of the curve C.

a) Find the equation on the straight line AB. **(1 mark)**

b) In the same Cartesian plane, draw the straight line AB and the curve C. **(3 marks)**

c) Calculate the area between the curve C and the straight line AB. **(6 marks)**

d) Calculate the volume of solid of revolution about the x-axis of the surface area in c) above. **(5 marks)**

19. a) Suppose f and g are linear transformations on real vector space IR^2 with their respective representative matrices $F=\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}$ and $G=\begin{bmatrix}3 & 0\\ -2 & 1\end{bmatrix}$ relative to the basis B. **(3 marks)**

Find the matrix that represents gof.

b) Find a vector u such that $f(u)=2u$ and vector v such that $f(v)=v$ **(4 marks)**

c) Prove that $B=(u,v)$ is a basis of the vector space IR^2 **(2 marks)**

d) Write the matrix T that represents f relative to the basis B. **(4 marks)**

e) Find a relationship between F and T. **(2 marks)**

20. a) For what point of the parabola $y^2=18x$, is the ordinate equal to three times the abscissa? **(5 marks)**

b) S and T are the foci of the ellipse $\frac{x^2}{a}+\frac{y^2}{b^2}=1$ and B is the end of the minor axis. If STB is an equilateral triangle, find the eccentricity of that ellipse. **(5 marks)**

c) A variable circle passes through a fixed point $(2,0)$ and touches the y-axis. Find the locus of its centre. **(5 marks)**